Maximum Likelihood (ML) (factor analysis algorithms)

Version 21.0.0 ▼

The maximum likelihood solutions of Λ and ψ^2 are obtained by minimizing

$$F = tr[(\Lambda\Lambda' + \psi^2)^{-1}\mathbf{R}] - \log|(\Lambda\Lambda' + \psi^2)^{-1}\mathbf{R}| - p$$

with respect to Λ and ψ, where *p* is the number of variables, Λ is the factor loading matrix, and ψ^2 is the diagonal matrix of unique variances.

The minimization of *F* is performed by way of a two-step algorithm. First, the conditional minimum of *F* for a given y is found. This gives the function $f(\psi)$, which is minimized numerically using the Newton-Raphson procedure. Let $\mathbf{x}^{(s)}$ be the column vector containing the logarithm of the diagonal elements of y at the *s*th iteration; then

$$\mathbf{x}^{(s+1)} = \mathbf{x}^{(s)} - \mathbf{d}^{(s)}$$

where $\mathbf{d}^{(s)}$ is the solution to the system of linear equations

$$\mathbf{H}^{(s)}\mathbf{d}^{(s)} = \mathbf{h}^{(s)}$$

and where

$$\mathbf{H}^{(s)} = (\partial^2 f(\psi)/\partial x_i \partial x_j)$$

and $\mathbf{h}^{(s)}$ is the column vector containing $\partial f(\psi)/\partial x_i$. The starting point $\mathbf{x}^{(1)}$ is

$$\mathbf{x}^{(1)} = \begin{cases} \log[(1 - m/2p)/r^{ii}] & \text{for ML and GLS} \end{cases}$$

$$x_i = \begin{cases} \quad \\ [(1-m/2p)/r^{ii}]^{1/2} & \text{for ULS} \end{cases}$$

where m is the number of factors and r^{ii} is the ith diagonal element

The values of $f(\psi)$, $\partial f/\partial x_i$, and $\partial^2 f/\partial x_i \partial x_j$ can be expressed in terms of the eigenvalues

$$\gamma_1 \le \gamma_2 \le \dots \le \gamma_p$$

and corresponding eigenvectors

$$\omega_1, \ \omega_2, \ \dots, \ \omega_p$$

of matrix $\psi \mathbf{R}^{-1} \psi$. That is,

$$f(\psi) = \sum_{k=m+1}^{p} \left(\log \gamma_k + \gamma_k^{-1} - 1 \right)$$

$$\frac{\partial f}{\partial x_i} = \sum_{k=m+1}^{p} \left(1 - \gamma_k^{-1} \right) \omega_{ik}^2$$

$$\frac{\partial^2 f}{\partial x_i \partial x_j} = -\delta_{ij} \frac{\partial f}{\partial x_i} + \sum_{k=m+1}^{p} \omega_{ik} \omega_{jk} \left(\sum_{n=1}^{m} \frac{\gamma_k + \gamma_n - 2}{\gamma_k - \gamma_n} \omega_{in} \omega_{jn} + \delta_{ij} \right)$$

where

$$\delta_{ij} = \begin{cases} 1 & \text{if } i = j \\ 0 & \text{if } i \ne j \end{cases}$$

The approximate second-order derivatives

$$\frac{\partial^2 f}{\partial x_i \partial x_j} \cong \left(\sum_{k=m+1}^{p} \omega_{ik} \omega_{jk} \right)^2$$

are used in the initial step and when the matrix of the exact second- ... nts of the vector $\mathbf{d}$ are greater than 0.1. If $\partial^2 f / \partial x_i^2 < 0.05$ (Heywood variables ... e elements of that column and row are set to 0. If the value of $f(\psi)$ is not decr ... the value of $f(\psi)$ decreases or 25 halvings fail to produce a decrease. (In this ... es until the largest absolute value of the elements of $\mathbf{d}$ is less than the criterion value (default 0.001) or until the maximum number of iterations (default 25) is reached. Using the converged value of ψ (denoted by

$\hat{\psi}$

), the eigenanalysis is performed on the matrix

$\hat{\psi}$
$\mathbf{R}^{-1}$

$\hat{\psi}$

. The factor loadings are computed as

$$\hat{\Lambda}_m = \hat{\psi}\, \Omega_m \left(\Gamma_m^{-1} - \mathbf{I}_m \right)^{1/2}$$

where

$$\Gamma_m = \text{diag}\,(\gamma_1, \gamma_2, \dots, \gamma_m)$$
$$\Omega_m = (\omega_1, \omega_2, \dots, \omega_m)$$