

On Balanced k -coverage in Visual Sensor Networks

Md. Muntakim Sadik¹, Sakib Md. Bin Malek², Ashikur Rahman³

^{1,2,3}Department of Computer Science and Engineering,

Bangladesh University of Engineering and Technology, Dhaka.

Email: ¹0905003.mms@ugrad.cse.buet.ac.bd, ²0905039.smbm@ugrad.cse.buet.ac.bd, ³ashikur@cse.buet.ac.bd

Abstract—Given a set of directional visual sensors, the k -coverage problem determines the orientation of minimal directional sensors so that each target is covered at least k times. As the problem is NP-complete, a number of heuristics have been devised to tackle the issue. However, the existing heuristics provide imbalance coverage of the targets—some targets are covered k times while others are left totally uncovered or singly covered. The coverage imbalance is more serious in under-provisioned networks where there do not exist enough sensors to cover all the targets k times. Therefore, we address the problem of covering each target at least k times in a balanced way using minimum number of sensors. We study the existing Integer Linear Programming (ILP) formulation for single coverage and extend the idea for k -coverage. However, the extension does not balance the coverage of the targets. We further propose Integer Quadratic Programming (IQP) and Integer Non-Linear Programming (INLP) formulations that are capable of addressing the coverage balancing. As the proposed formulations are computationally expensive, we devise a faster Centralized Greedy k -Coverage Algorithm (CGkCA) to approximate the formulations. Finally, through rigorous simulation experiments we show the efficacy of the proposed formulations and the CGkCA.

I. INTRODUCTION

A visual sensor network (VSN), also known as a Smart Camera Network (SCN), consists of a set of targets to be monitored by a set of *smart* (visual) sensors capable of self-controlling their orientations and ranges. Such VSNs have drawn considerable attention of researchers due to their enormous applicability in real-world scenarios like surveillance system, environment monitoring, smart traffic controlling system etc., to name a few.

The primary goal of VSNs is to monitor as many targets as possible [5], [9]. However, if the sensor covering a target malfunctions, runs out of power, or if the line of sight is blocked by a perpetrator, a previously covered target may suddenly become uncovered. The simplest solution to this problem is to incorporate *fault tolerance* besides *coverage*, i.e., to cover the target by more than one sensor. This joint *fault tolerant coverage* problem is well-known as “ k -coverage” problem in the literature.

Formally, our work tackles the k -coverage problem, where each target is to be covered by at least k sensors ($k \geq 1$). The efficiency of the solution to the problem depends on the extent of camera usage as fewer number of active sensors implies lower energy consumption and longer network life time. Thus, in k -coverage problem one needs to minimize camera-usage besides covering each target in a fault-tolerant way.

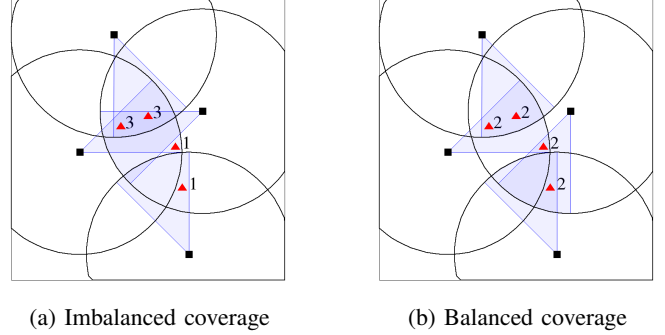


Fig. 1: Coverage imbalance in under-provisioned systems

We envision two kinds of visual sensor networks:—(i) under-provisioned networks and (ii) over-provisioned networks. We call a VSN is under-provisioned if the number of sensors is insufficient to cover all the targets at least k times and over-provisioned otherwise. Consider Fig. 1(a) for an example of an under-provisioned network. In this figure, there are 4 cameras (rectangular ones) and 4 targets (triangular ones) and the objective is to cover each target at least 3 times (i.e., $k = 3$). Here, every camera possesses a specific number of non-overlapping pans, of which only one can be selected in a particular deployment and each pan is defined by a field of view (FoV) angle, $\theta = \frac{\pi}{4}$. Thus, a camera can pick any one of eight ($\frac{2\pi}{\theta} = \frac{2\pi}{\frac{\pi}{4}} = 8$) disjoint pans/orientations. Note that, no orientation of 4 cameras can produce a 3-coverage for this scenario, hence the network is consequently called “under-provisioned”.

When it is impossible to provide a complete k -coverage for an under-provisioned network, we need to provide a more *fault resilient* solution instead. For example, let’s re-consider the under-provisioned network in Fig. 1(a) and Fig. 1(b). Although both the figures show the same deployment of cameras and targets but the orientation of the cameras are different. In Fig. 1(a) the targets are covered by 3, 3, 1, and 1 sensors respectively, while in Fig. 1(b), each of the targets are covered by 2 sensors. However, the coverage in Fig. 1(b) is more *fault resilient* than the coverage in Fig. 1(a) because in Fig. 1(a) two targets are covered once, while in Fig. 1(b) all the targets are covered twice. We say the coverage in Fig. 1(b) is more *balanced* than coverage in Fig. 1(a).

In this paper, we focus on under-provisioned systems and provide solutions for *coverage balancing* using minimum number of sensors. The problem is eventually an instance of the classical set *multi-cover* problem whose optimization

version is known to be NP-hard [1]. In order to define *coverage balancing*, we borrow the concept of *fairness* in resource allocation systems proposed by Jain et al. [10] and modify it for our purpose (the modification is described in a later section). The *Fairness Index*, \mathcal{FI} , is defined as follows [10]. Suppose we have $1, 2, 3, \dots, m$ components in a system and x_i is the resource allocated to the i^{th} component. The fairness index of such system will be:

$$\mathcal{FI} = \frac{(\sum x_i)^2}{m \sum x_i^2} \quad (1)$$

In VSN, the sensors are the resources and the targets are the components of the system. Therefore, we can use the fairness index to judge by how much a system is balanced. Here, the number of times each target is covered is considered as the number of resources allocated to that target. Thus, the fairness index value of Fig. 1(a) is:

$$\mathcal{FI} = \frac{(3 + 3 + 1 + 1)^2}{4 \times (3^2 + 3^2 + 1^2 + 1^2)} = 0.8$$

and, for Fig. 1(b) is:

$$\mathcal{FI} = \frac{(2 + 2 + 2 + 2)^2}{4 \times (2^2 + 2^2 + 2^2 + 2^2)} = 1.0$$

Consequently, the camera orientations in Fig. 1(b) has more coverage balancing than the camera orientations in Fig. 1(a).

The major contributions of this paper are as follows:

(i) We introduce a novel *balanced k-coverage problem* for visual sensor networks (VSNs).

(ii) We study the existing exact Integer Linear Programming (ILP) formulation for single coverage and extend the idea for k -coverage. Then we show that this natural extension provides imbalance coverage of the targets in a sense that some targets are covered k -times while some targets are left totally uncovered or singly covered. The imbalance is more serious in *under-provisioned* networks.

(iii) We propose a novel Integer Quadratic Programming (IQP) formulation that improves fairness by balancing coverage while trying to achieve k -coverage. We further improve the fairness by providing another novel Integer Non-Linear Programming (INLP) formulation.

(iv) The proposed ILP, IQP, and INLP formulations are computationally expensive. Therefore, we formulate a novel computationally faster Centralized Greedy k -Coverage Algorithm (CG k CA) to approximate our formulations. Finally, we measure the relative performance of our formulations and the CG k CA algorithm in terms of coverage balancing.

The road map of the paper is as follows. This section introduces the coverage balancing problem and the motivation behind this work. Section II introduces the description and parameters of a Visual Sensor Network and formally defines the problem that we solve in this paper. Section II also discusses the shortcomings of Fairness Index in capturing coverage balance and introduces a new metric Balancing Index. Section III shows how to formulate Integer Linear Programming (ILP) for the k -coverage problem and discusses

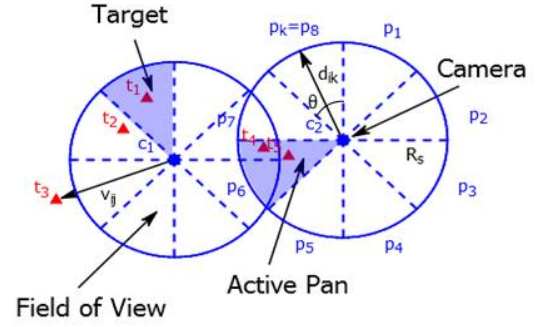


Fig. 2: Camera coverage parameters

ILP's incapability in coverage balancing. Section IV modifies ILP to formulate Integer Quadratic Programming (IQP) and Integer Non-Linear Programming (INLP) that incorporate coverage balancing besides k -coverage. Section V discusses the Centralized Greedy k -Coverage Algorithm (CG k CA). Section VI presents the simulation results and analyzes the results. Section VII provides a brief literature review on the subject matter and finally Section VIII concludes the paper. Throughout the paper we use the terms "camera" and "sensor" interchangeably.

II. PRELIMINARIES

In this section we formally introduce the VSN with relevant parameters and provide a formal description of the problem.

A. Visual Sensor Network Description and Parameters

The sensing region of a camera can be characterized by its Field of View (FoV) which is defined as the extent of the observable/sensing region that can be captured at any given direction. Some cameras come with fixed-FoV and for some, FoVs are adjustable. The smart cameras used in current VSNs are known as *Pan-Tilt-Zoom* (PTZ) cameras where FoV can be self-adjusted in three dimensions: (i) horizontal movement in pan, (ii) vertical movement or tilt, and (iii) change in depth-of-field by changing zoom. In this paper, we limit ourselves to pan-only cameras, i.e., we assume that a camera can move only in horizontal direction and its FoV is only described by its pan. The pan of a camera is formally defined using the following two parameters:

- R_s : Maximum coverage *range* of the camera beyond which a target can not be detected with acceptable accuracy in a binary detection test.
- θ : The maximum sensing/coverage *angle* of a camera on a certain direction. This angle is also known as Angle of View (AoV).

Thus, when a camera is oriented towards a particular direction, it can cover a circular sector (called a pan) defined by R_s and θ . We assume that every camera possesses a specific number of non-overlapping pans, of which, only one can be selected in a particular deployment. For example: a camera with FoV defined by $\theta = \frac{\pi}{4}$ can pick any one of eight disjoint orientations. Fig. 2 depicts these parameters of camera coverage. Here, two cameras c_1 and c_2 have eight

pans each and can be oriented towards any of these eight pans. We assume that cameras are homogeneous in terms of parameters. Position of a target and a sensor are expressed through Cartesian coordinates (x, y) in a two-dimensional plane. \vec{d}_{ij} is a unit vector which cuts each pan (i.e., the sensing sector) into half representing the orientation of camera c_i towards pan p_j . \vec{v}_{it} is a vector in the direction from camera c_i to target g_t .

Target in Sector (TIS) Test: With TIS test [1], one can verify whether a target g_t is coverable by a given sensor s_i . To conduct this test, at first we calculate the angle ϕ_{it} between camera orientation \vec{d}_{ij} of pan p_j and the target vector \vec{v}_{it} .

$$\phi_{it} = \cos^{-1} \left(\frac{\vec{d}_{ij} \cdot \vec{v}_{it}}{|\vec{d}_{ij}| |\vec{v}_{it}|} \right) \quad (2)$$

A target is coverable by a camera's FoV if the span of its FoV contains the target and the target is located within the sensing range of the camera. Geometrically, \vec{d}_{ij} divides the pan p_j into two equal halves and if a target is located in either of them, it is coverable by that camera on the pan p_j . Thus, the angle ϕ_{it} needs to be less than half of the AoV, i.e., $\phi_{it} \leq \frac{\theta}{2}$. The other condition requires that the target has to be inside the maximum sensing range of the camera, i.e., $|v_{it}| \leq R_s$.

Conducting TIS tests over every pan p_j of camera s_i and every target g_t , we can build a *binary coverage matrix* $A_{N \times Q}^M$ of the network comprising of M targets and N cameras with Q pans where an entry in the matrix can be calculated as:

$$a_{ij}^t = \begin{cases} 1 & \text{if target } g_t \text{ is covered by camera } s_i \text{ at pan } p_j \\ 0 & \text{otherwise} \end{cases} \quad (3)$$

B. Fairness Index and Balancing Index in k -coverage

Although the objective of the k -coverage problem is to cover each target at least k times, it may not be achievable if enough sensors are not available (i.e., in under-provisioned networks). Therefore, for under-provisioned networks a new metric needs to be defined to determine the superiority of one solution to another. The *Fairness index* proposed by Jain et al. [10] has been traditionally used to measure *fairness* in resource allocation systems. The visual sensor network can be thought of a similar resource allocation system where the sensors are the resources and the targets are the components of the system. Suppose in a particular configuration and orientation of sensors, the m targets are covered $\psi_1, \psi_2, \dots, \psi_m$ times respectively. Then the merit of this *solution vector* $(\psi_1, \psi_2, \dots, \psi_m)$ can be measured using the following equation:

$$\mathcal{FI} = \frac{(\sum_{t=1}^m \psi_t)^2}{m \sum_{t=1}^m \psi_t^2} \quad (4)$$

Thus, the fairness index of two solution vectors $(3, 3, 1, 1)$ and $(2, 2, 2, 2)$ each of which tries to cover four targets in a 3-coverage problem is 0.8 and 1.0 respectively which shows the solution $(2, 2, 2, 2)$ is superior than $(3, 3, 1, 1)$.

Even though fairness index is a good performance metric, solely focusing on maximizing it can result in reducing the

total coverage. Fairness index usually identifies fairer solutions in the allocation of resources. Thus, in a 3-coverage problem with 3 targets, $(2, 2, 2)$ coverage is fairer than $(2, 3, 2)$ although the second one is preferred because 3-coverage has not been attained at all in the solution $(2, 2, 2)$ for any of the targets.

Therefore, we modify the concept of fairness index to fit in to our purpose and introduce a more suitable metric called *Balancing Index*, \mathcal{BI} , which combines both fairness of coverage and maximization of the total coverage. It is defined as the product of fairness index and the ratio of achieved total coverage (i.e., $\sum_{t=1}^m \psi_t$) over total attainable coverage (i.e., km). Mathematically, the balancing index is:

$$\mathcal{BI} = \mathcal{FI} \times \frac{\sum_{t=1}^m \psi_t}{km} = \frac{(\sum_{t=1}^m \psi_t)^2}{m \times \sum_{t=1}^m \psi_t^2} \times \frac{\sum_{t=1}^m \psi_t}{km} \quad (5)$$

where ψ_t is the number of sensors covering target g_t and m is the total number of targets. The balancing index value for $(2, 2, 2)$ is:

$$\mathcal{BI} = \frac{(2 + 2 + 2)^2}{3 \times (2^2 + 2^2 + 2^2)} \times \frac{(2 + 2 + 2)}{3 \times 3} = 0.6666$$

The balancing index value for $(2, 3, 2)$ is:

$$\mathcal{BI} = \frac{(2 + 3 + 2)^2}{3 \times (2^2 + 3^2 + 2^2)} \times \frac{(2 + 3 + 2)}{3 \times 3} = 0.7472$$

The balancing index reflects that $(2, 3, 2)$ coverage is better than $(2, 2, 2)$ coverage in a 3-coverage problem. Thus, it can be used as the performance metric. The higher the value of the balancing index, the better is the coverage.

C. Problem Formulation

The balanced k -coverage problem can formally be described as follows:

Given: A set of targets, $\mathcal{T} = \{g_1, g_2, \dots, g_m\}$ to be covered; a set of homogeneous directional sensors, $\mathcal{S} = \{s_1, s_2, \dots, s_n\}$, each of which can be oriented in one direction of q possible non-overlapping pans; the pan set, $\mathcal{P} = \{p_1, p_2, \dots, p_q\}$. A collection of subsets, $\mathcal{F} = \{\Phi_{\langle i, j \rangle} | s_i \in \mathcal{S}, p_j \in \mathcal{P}\}$ can be generated based on a TIS test, where $\Phi_{\langle i, j \rangle}$ is a subset of \mathcal{T} and denotes the set of targets covered by selecting sensor s_i and oriented in the direction of pan p_j .

Problem: Find a sub-collection \mathcal{Z} of \mathcal{F} , with the constraint that at most one $\Phi_{\langle i, j \rangle}$ can be chosen for the same sensor s_i , and the Balancing Index, \mathcal{BI} (defined in Equation 5), gets maximized.

III. GENERIC K -COVERAGE AND ITS ILP FORMULATION

In [1], the authors elaborate an ILP formulation to solve the maximization of *single* coverage using minimum number of sensors (MCMS). The proposed ILP formulation can be easily extended for the k -coverage problem. In this section, at first we describe the necessary modifications and then we point out its shortcomings in providing a balanced solution for under-provisioned networks.

The parameters used for the formulation can be summarized as follows. n : the number of sensors; m : the number of targets;

q : the number of orientation available for each directional sensor. The variables in the formulation are as follows. ψ_t : an integer variable that has a value equal to the number of times a target g_t is covered by directional sensors, limited up to a maximum value k ; $\chi_{\langle i,j \rangle}$: a binary variable that has value one if the directional sensor s_i uses the orientation p_j , and zero otherwise; ξ_t : an integer variable that counts the number of the directional sensors covering target g_t . $\Phi_{\langle i,j \rangle}$ is the set of targets that are covered by the sensor s_i in its pan p_j . Using TIS, for each sensor s_i , incidence matrix $A_{(m \times q)}^i$ can be generated, where each of its elements would be:

$$a_{ij}^t = \begin{cases} 1 & t \in \Phi_{\langle i,j \rangle} \\ 0 & \text{otherwise} \end{cases} \quad (6)$$

Therefore, ξ_t can be expressed as:

$$\xi_t = \sum_{i=1}^n \sum_{j=1}^p a_{ij}^t \chi_{\langle i,j \rangle}$$

Now, the ILP formulation for k -coverage problem becomes:

$$\text{maximize} \sum_{t=1}^m \psi_t - \rho \sum_{i=1}^n \sum_{j=1}^q \chi_{\langle i,j \rangle} \quad (7)$$

subject to:

$$\frac{\xi_t}{n} \leq \psi_t \leq \xi_t \quad \forall t = 1 \dots m \quad (8)$$

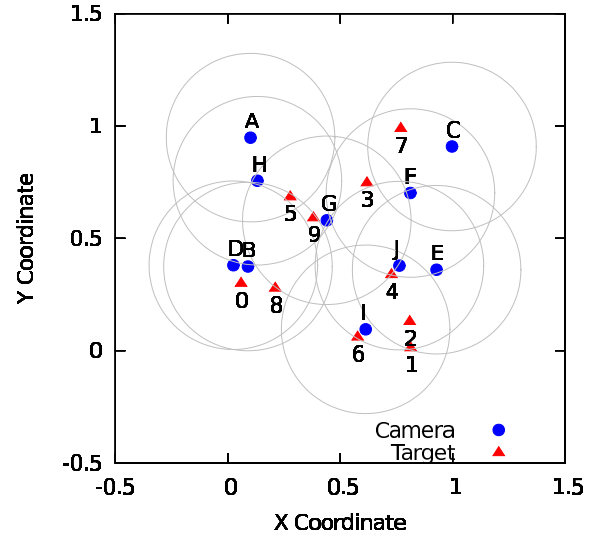
$$\psi_t \leq k \quad (9)$$

$$\sum_{j=1}^q \chi_{\langle i,j \rangle} \leq 1 \quad \forall i = 1 \dots n \quad (10)$$

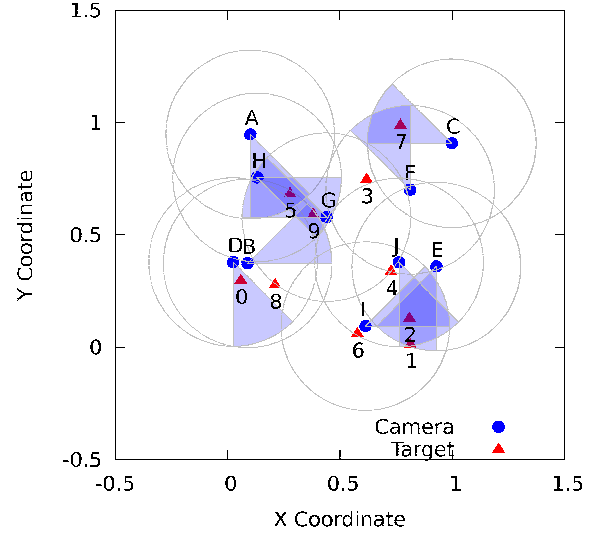
$$\chi_{\langle i,j \rangle} = 0 \quad \text{or} \quad 1 \quad \forall i = 1 \dots n, \quad \forall j = 1 \dots q \quad (11)$$

The objective function defined by Equation 7 maximizes the coverage count of each target and imposes a penalty by multiplying the number of sensors to be activated by a small positive penalty coefficient ρ (≤ 1). There are $(m + np)$ variables and $(2m + n + np)$ constraints for the ILP. Equation 8 represents a set of inequalities to indicate whether any target g_t is covered or not: if none of the sensors cover target g_t , i.e., $\xi_t = 0$, then $\psi_t = 0$ to conform the right inequality; if target g_t is covered by at least one directional sensor, i.e., $\xi_t > 0$, since ξ_t is bounded by n , ξ_t/n is a real number less than one, then $\psi_t \geq 1$ to follow the left inequality. Constraints in Equation 9 make sure that the coverage count of any target is bounded by k , i.e., even if a target is covered by more than k times still the coverage count will be considered as k , no additional benefit for covering a target more than k times. Equation 10 guarantees that one directional sensor has at most one orientation depending on whether it is activated or not.

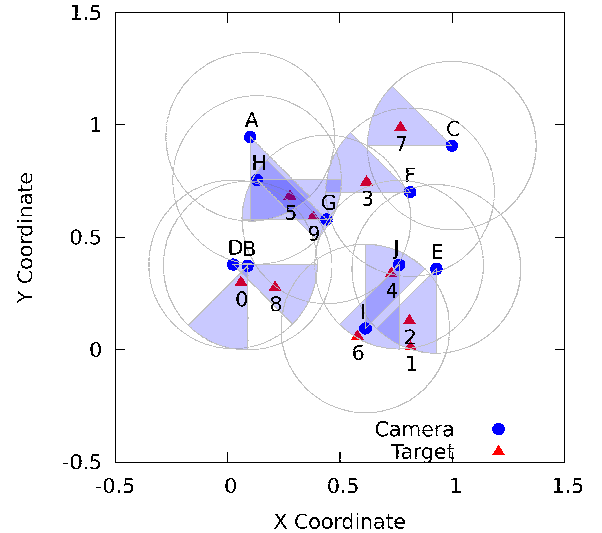
Problem with generic ILP: The ILP formulation mentioned above for k -coverage problem does not focus on coverage balancing. To understand the problem, consider the scenario shown in Fig. 3(a). Here we have shown a 3-coverage problem with 10 targets (red triangles) and 10 directional sensors (blue circles). FoV is defined by $\theta = \frac{\pi}{4}$. Table I summarizes the



(a) Initial Configuration 10 Targets and 10 Cameras



(b) Pan selection of each camera in ILP



(c) Pan selection in balanced-optimal configuration

Fig. 3: Illustrating imbalance coverage of ILP formulation

TABLE I: Detail analysis of example scenario in Fig. 3

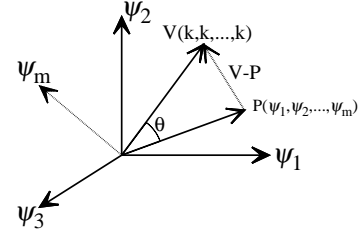
| Target ID | Maximum possible coverage | Coverage achieved in ILP | Coverage achievable in Balanced-optimal |
|-----------|---------------------------|--------------------------|---|
| 0 | 2 | 1 | 1 |
| 1 | 3 | 2 | 1 |
| 2 | 3 | 3 | 1 |
| 3 | 2 | 0 | 1 |
| 4 | 5 | 0 | 2 |
| 5 | 4 | 3 | 3 |
| 6 | 2 | 0 | 1 |
| 7 | 2 | 2 | 1 |
| 8 | 3 | 0 | 1 |
| 9 | 3 | 3 | 2 |

coverage counts under different conditions. Not all targets are 3-coverable:—it is possible to cover the targets $\{0, 3, 6, 7\}$ at most twice. Rest of the targets are at least 3-coverable. Clearly the network is under-provisioned. After running the ILP, formulated above, we found a solution which is shown in Fig. 3(b). The third column of Table I captures the coverage achieved in ILP. Targets $\{2, 5, 9\}$ are covered thrice, targets $\{1, 7\}$ are covered twice, target $\{0\}$ is singly covered and noticeably targets $\{3, 4, 6, 8\}$ are left totally uncovered.

In summary, 40% targets are not covered by any of the sensors in the solution provided by the ILP. Fig. 3(c) shows another possible solution of the same problem which we call *balanced optimal* coverage (describe in Section IV-C). The fourth column of Table I shows the coverage achieved in this new solution. Only target $\{5\}$ and targets $\{2, 9\}$ are 3-covered and 2-covered respectively however the rest of all targets are singly covered. Unlike ILP, none of the targets are left uncovered in this new solution.

IV. BALANCED K-COVERAGE

In order to improve the balancing of coverage, we need to modify the ILP formulation, specially its objective function. In particular, the objective function concurrently needs to keep track of balancing of coverage while pursuing k -coverage. To keep the problem tractable, we may consider the solutions of k -coverage problem as vectors in an m -dimensional vector space. The coverage counts of targets can be considered as individual dimensions in the m -dimensional space. Fig. 4 shows this typical scenario. Each axis represents the coverage count of each target and there is a total of m possible targets. The vector $\mathbf{V} \equiv (k, k, \dots, k)$ represents the desired solution vector. Let us consider the vector \mathbf{P} which represents the achieved coverage by an arbitrary algorithm. Hence vector \mathbf{P} can be represented as $\mathbf{P} \equiv (\psi_1, \psi_2, \dots, \psi_k)$. Although it is highly desirable to align the vector \mathbf{P} with the vector \mathbf{V} , but practically it may not be achieved by an algorithm. In such cases, the goal should be to *minimize* the *distance* between these two vectors, $\overrightarrow{P\mathbf{V}}$ [$\overrightarrow{P\mathbf{V}} = \mathbf{V} - \mathbf{P}$] or to *minimize* the *angle* between them (i.e., the θ). We formally describe both of these intuitive approaches below.

Fig. 4: Coverage displayed in m -dimensional space

A. Minimizing the vector distance

The vector distance between the actual coverage vector $\mathbf{P}(\psi_1, \psi_2, \dots, \psi_k)$ and the expected coverage vector $\mathbf{V}(k, k, \dots, k)$ can be calculated as follows:

$$d(\mathbf{V}, \mathbf{P}) = \|\mathbf{V} - \mathbf{P}\| = \sqrt{\sum_{t=1}^m (k - \psi_t)^2}$$

where k is the number of times the targets are to be covered and ψ_t is the achieved coverage of target g_t . The minimization problem remains the same even if we ignore the square root. Thus, the ILP formulation described in Section III can be easily modified to achieve the goal. We can simply modify the objective function (Equation 7) and incorporate the square of the vector distance leaving all constraints (Equations 8 - 11) unchanged. The new objective function will be as follows:

$$\text{minimize} \quad \sum_{t=1}^m (k - \psi_t)^2 + \rho \sum_{i=1}^n \sum_{j=1}^q \chi_{(i,j)}$$

The modified objective function is no more linear but quadratic in nature. Therefore, the new formulation is an *integer quadratic programming* problem or IQP in short.

B. Minimizing the angle

Another approach is to minimize the angle between the two vectors \mathbf{V} and \mathbf{P} using the following equation:

$$\theta = \cos^{-1} \left(\frac{\mathbf{V} \cdot \mathbf{P}}{\|\mathbf{V}\| \|\mathbf{P}\|} \right) = \cos^{-1} \left(\frac{\sum_{t=1}^m \psi_t}{\sqrt{\sum_{t=1}^m \psi_t^2}} \right)$$

However, minimizing the angle between vectors can not differentiate isomorphic solutions like $(1, 1, \dots, 1)$, $(2, 2, \dots, 2)$, $(3, 3, \dots, 3)$... etc. because all of these vectors make an angle zero with the ideal solution vector (k, k, \dots, k) . Therefore, we do not explore this issue further in this paper.

C. Maximizing the balancing index

Finally, the objective function of the ILP can be modified to incorporate **balancing index** as defined in Section II-B and achieve true coverage balancing. The necessary modification is as follows:

$$\text{maximize} \quad \frac{1}{km^2} \times \frac{(\sum_{t=1}^m \psi_t)^3}{\sum_{t=1}^m \psi_t^2} - \rho \sum_{i=1}^n \sum_{j=1}^p \chi_{(i,j)}$$

subjected to the constraints described in Equations 8 - 11. Note that, the new objective function is non-linear in nature. Therefore, the modified formulation falls within the domain of *integer non-linear programming* problems or INLP in short.

Algorithm 1 Centralized Greedy k -coverage Algorithm (CG k CA)

Input: $\Phi_{\langle i,j \rangle}$ {set of targets covered by sensor s_i in pan p_j }
Output: \mathcal{Z} {a collection of ⟨active sensor, orientation⟩ pairs}

- 1: $\mathcal{Z} \leftarrow \emptyset$
- 2: $\mathcal{Y} \leftarrow$ {set of inactive nodes}
- 3: $\mathcal{T} \leftarrow \{g_1, g_2, \dots, g_m\}$ {set of all targets}
- 4: $\mathcal{C} \leftarrow \{c_1, c_2, \dots, c_m\}$ { c_i is the coverage count of g_i }
- 5: **repeat**
- 6: $maxincentive \leftarrow 0$
- 7: $\alpha \leftarrow 0$ for linear benefit; $\alpha \leftarrow 1$ for quadratic benefit
- 8: **for** $\forall i \in \mathcal{Y}$ **do**
- 9: **for** $\forall j \in \mathcal{P}$ **do**
- 10: $incentive \leftarrow \text{BENEFIT}(\Phi_{\langle i,j \rangle}, \mathcal{Z}, \alpha)$
- 11: **if** $incentive > maxincentive$ **then**
- 12: $maxincentive \leftarrow incentive$
- 13: $\langle i^{max}, j^{max} \rangle = \langle i, j \rangle$
- 14: **end if**
- 15: **end for**
- 16: **end for**
- 17: $\mathcal{Z} \leftarrow \mathcal{Z} \cup \langle i^{max}, j^{max} \rangle$
- 18: $\mathcal{Y} \leftarrow \mathcal{Y} \setminus \{i^{max}\}$
- 19: **until** $maxincentive = 0$
- 20: **return** \mathcal{Z}

V. BALANCED k -COVERAGE HEURISTICS

The ILP, IQP, and INLP formulation of the problem mentioned in the previous section can be used to find the optimal solutions, however, they are not scalable in large problem instances. Therefore, we present Centralized Greedy k -coverage Algorithm (CG k CA), a polynomial time greedy heuristic that would converge faster and by suitably choosing appropriate set of sensors would also balance the coverage. CG k CA uses a *benefit function*, which calculates the incentive of selecting a particular ⟨sensor, pan⟩ pair at each step.

The basic idea of CG k CA is to greedily choose and activate the ⟨sensor, pan⟩ pair which provides the maximum benefit. In each iteration, the incentives of all inactive ⟨sensor, pan⟩ pairs are calculated using the benefit function. The pair with maximum incentive is selected and the sensor is activated towards the corresponding pan. Ties are broken arbitrarily by choosing among the pairs providing maximum incentive. The algorithm terminates when all the sensors are activated or when all the targets are at least k -covered. The pseudo code of CG k CA is given in Algorithm 1.

Benefit function: In order to choose from all inactive sensor-pan pairs, CG k CA makes a call to a benefit function in step 10 of Algorithm 1. The benefit function, which is defined in Algorithm 2, calculates two different kinds of benefit:–(i) linear benefit and (ii) quadratic benefit. The three parameters of the benefit function is as follows:

$\Phi_{\langle i,j \rangle}$: set of targets covered by sensor-pan pairs $\langle i, j \rangle$.

\mathcal{Z} : set of sensor-pan pairs activated so far by the greedy algorithm before this step.

Algorithm 2 Benefit Function for k -coverage

Input: $\Phi_{\langle i,j \rangle}$ {set of targets covered by sensor s_i in pan p_j },
 \mathcal{Z} {set of assigned ⟨active sensor, orientation⟩ pairs}, α
{ $\alpha = 0$ for linear benefit and $\alpha = 1$ for quadratic benefit}
Output: $incentive$ {an integer containing the total incentive for $\langle i, j \rangle$ for given \mathcal{Z} }

- 1: **function** BENEFIT($\Phi_{\langle i,j \rangle}, \mathcal{Z}, \alpha$)
- 2: $incentive \leftarrow 0$
- 3: **for** $\forall t \in \Phi_{\langle i,j \rangle}$ **do**
- 4: Calculate the coverage c_t of target t using \mathcal{Z}
- 5: **if** $c_t < k$ **then**
- 6: **if** $\alpha = 0$ **then**
- 7: $increment \leftarrow 1$
- 8: **else if** $\alpha = 1$ **then**
- 9: $increment \leftarrow (k - c_t)^2 - (k - c_t - 1)^2$
- 10: **end if**
- 11: $incentive \leftarrow incentive + increment$
- 12: **end if**
- 13: **end for**
- 14: **end function**
- 15: **return** $incentive$

α : a boolean parameter to indicate whether the benefit function should calculate either linear or quadratic benefit. $\alpha = 0$ means linear and quadratic otherwise. The linear benefit is calculated as follows. For each target g_t , let us define a variable c_t which assumes a value equal to the coverage count of that target by the set of sensor-pan pairs in set \mathcal{Z} , i.e.:

$$c_t = \text{number of sensing regions in } \mathcal{Z} \text{ that cover target } g_t$$

Then the total linear benefit of activating sensor-pan pair $\langle i, j \rangle$ at this stage of CG k CA is:

$$\sum_{t \in \Phi_{\langle i,j \rangle} \wedge c_t < k} \min\{1, k - c_t\}$$

Note that, the linear benefit function ignores the coverage count of a target at this stage of the greedy algorithm and assigns same incentive for covering a target irrespective of its coverage count. The quadratic benefit function eliminates this drawback and provides more incentive for covering a less covered target as opposed to highly covered targets. The incentive of covering a target is quadratic in nature and is defined by $k^2 - (k - i)^2$ for a target that is covered i times. Thus the quadratic benefit of activating sensor-pan pair $\langle i, j \rangle$

TABLE II: Incentive table for 3-coverage problem ($k = 3$)

| Coverage count of a target at this stage | Incentive in linear benefit function | Incentive in quadratic benefit function |
|--|--------------------------------------|---|
| c_t | $\min\{1, k - c_t\}$ | $(k - c_t)^2 - (k - c_t - 1)^2$ |
| 0 | 1 | 5 |
| 1 | 1 | 3 |
| 2 | 1 | 1 |

at this stage of $CGkCA$ becomes:

$$\sum_{t \in \Phi_{(i,j)} \wedge c_t < k} (k - c_t)^2 - (k - c_t - 1)^2$$

Table II shows the incentive values for both linear and quadratic benefit functions that will be rewarded while solving a 3-coverage problem. Algorithm 2 provides the complete pseudo-code of the benefit function. For rest of the paper, when the greedy algorithm runs with the linear benefit function, the approach is dubbed as *Greedy Linear*. Similarly, when it runs with quadratic benefit function, we call it *Greedy Quadratic*.

Time complexity: $CGkCA$ needs the incidence matrix as input. To generate the incidence matrix, we need to iterate over all the n sensors in each of their q pans and check for each target if they satisfy the TIS test. As the TIS test takes $O(1)$ time for a specific sensor in its specific pan and a fixed target, the whole generation would take $O(nmq)$ time.

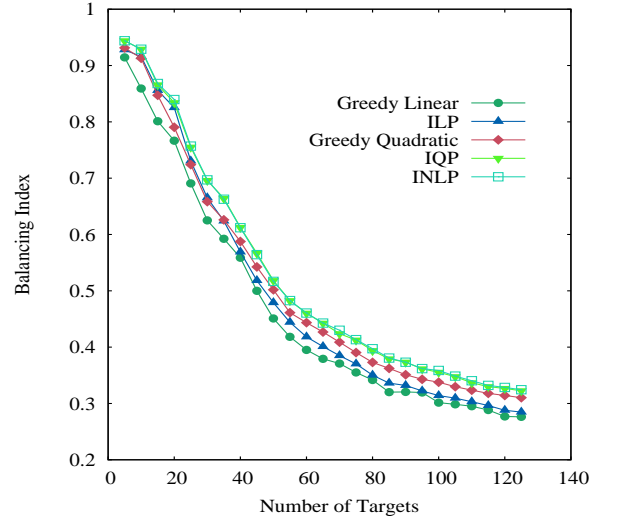
The major contributor to time complexity of the algorithm is due to the calculation of maximum incentive. The loop from line 5 to 20 is executed at most n times to check for each sensor. In each iteration, the benefit function will be called $O(nq)$ times. On each call, benefit function will check all the target within that pan and calculate the coverage counts of those targets and then the incentive of the $\langle \text{sensor}, \text{pan} \rangle$ pair. It will cost $O(mn)$. Therefore, the cost of each iteration is $O(n^2qm)$. Thus, the overall time complexity of the algorithm becomes $O(n^3qm)$.

VI. EXPERIMENTAL RESULTS

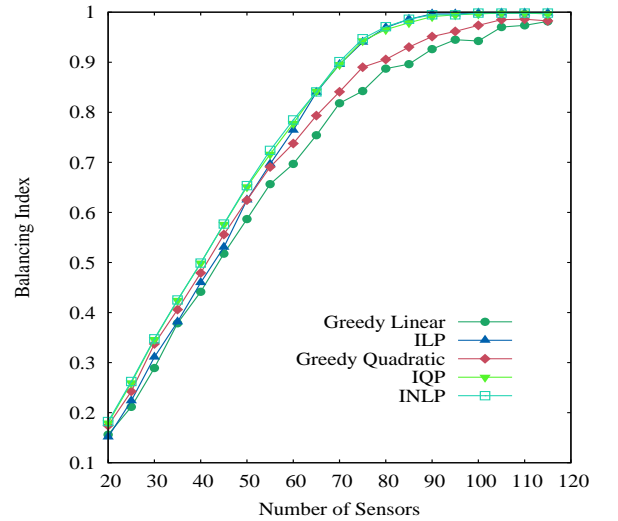
In order to verify and compare the effectiveness of proposed ILP, IQP, and INLP formulations and the greedy algorithm with linear and quadratic benefit functions, we perform rigorous simulation experiments. We use *balancing index*, \mathcal{BI} , defined in Section II-B as the performance metrics of comparison. A higher value of \mathcal{BI} indicates highly balanced coverage. At the end, we also comment on the sensor usage by the different formulations and greedy heuristics.

A. Simulation Environment

The deployment area is modelled as a 2D grid where targets are considered points in the grid and the sensors are modelled as directional sensors with FoV, $\theta = \frac{\pi}{4}$. We run two different types of experiments. In one kind, we keep the number of sensors fixed at 50 and vary the number of targets from 5 to 125 and in another kind, we keep the number of targets fixed at 50 and vary the number of available sensors from 20 to 115. For both scenarios the sensing range $R_s = 25$ units and the grid size is 125×125 sq. units. In both cases, the scenarios are generated in such a way that the smaller scenario is a subset of a larger scenario. This ensures a consistent evaluation of the impact of the enlarged population of sensors/targets by retaining all the “features” of the previous environment and simply making it better/worse.



(a) Performance comparison while varying the number of targets; the number of sensors is fixed at 50



(b) Performance comparison while varying the number of sensors, the number of targets is fixed at 50

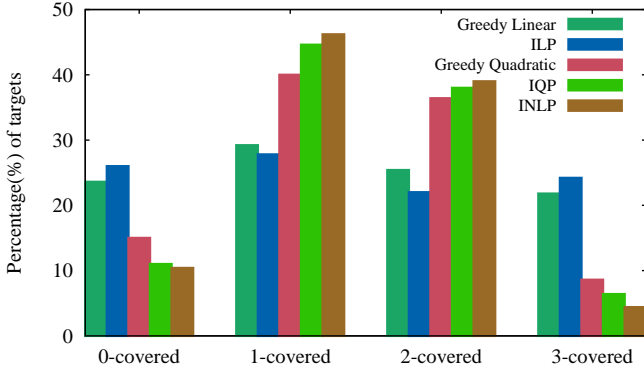
Fig. 5: Effect on Balancing Index

B. Performance comparison of different approaches

We capture performance of the proposed approaches by measuring \mathcal{BI} while changing the network’s state from under-provisioned to over-provisioned type and vice-versa.

In the first type of environment, when we increase the number of targets gradually from 5 to 125 keeping number of sensors fixed at 50, the network’s state changes from over-provisioned to under-provisioned as the number of targets slowly overwhelms the number of sensors. As a result, from Fig. 5(a) it is clearly evident that the curves move farther away from the ideal coverage ($\mathcal{BI} = 1$). In the whole downward progress, INLP and IQP clearly outperform all other methods.

In the second type of environment, increasing the number of available sensors and keeping the number of targets fixed at 50 shifts the state of the network from under-provisioned to over-provisioned. This behaviour is reflected in all curves



Number of sensors = 50, number of targets = 100

Fig. 6: Coverages in Under-provisioned Network

as there is an upward movement toward the ideal coverage in Fig. 5(b). Also here, the performance of INLP and IQP formulations exceeds the other approaches.

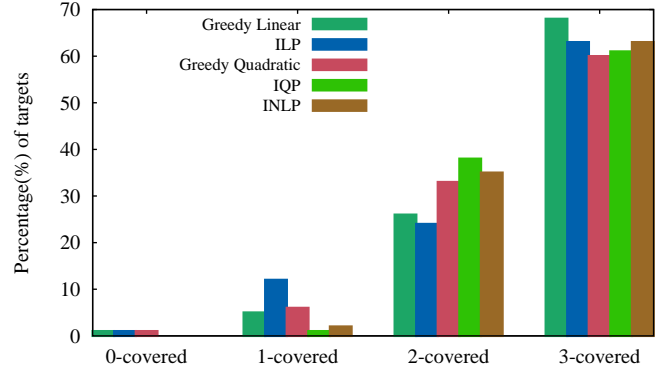
In both cases, the Greedy Quadratic shows greater coverage and balancing than the ILP formulation in under-provisioned condition, however when the network starts to contain larger number of sensors relative to the number of targets, ILP crosses the Greedy Quadratic curve. In a completely over-provisioned scenario, ILP almost merges with IQP and INLP formulations. Greedy Linear fails to keep up with all other formulations in all cases. One notable point is that, IQP curve is almost merged with INLP curves throughout the whole simulation but never exceeds the performance of INLP. We can conclude that the Greedy Quadratic approximates the optimal behaviour very closely and with a very reasonable amount of computational time.

C. Coverage Analysis

Now we present a detailed coverage analysis of both under-provisioned and over-provisioned networks.

1) *Under-provisioned Networks*: We have deliberately created an under-provisioned network by keeping the number of targets to 100 and the number of sensors to 50. Clearly there are not enough sensors available to cover all the targets k times ($k = 3$). The detailed coverage analysis of such network is shown in Fig. 6. In this under-provisioned network, INLP as well as IQP tries to reduce the number of uncovered targets at the expense of number of higher covered targets. The percentage of uncovered targets is 10.4% and 11% for INLP and IQP respectively. The Greedy Quadratic roughly approximates this behaviour by reducing the number of uncovered targets to 15%. The other two approaches (ILP and Greedy Linear) do not focus on coverage balancing and as a result they increase the coverages of some targets, keeping a large number of targets totally uncovered.

2) *Over-provisioned Network*: Next, we have created an over-provisioned network with 50 sensors and 20 targets. Clearly this is an over-provisioned network since there are enough available sensors to cover most of the targets at least k times. Again the detailed coverage analysis of such network



Number of sensors = 50, number of targets = 20

Fig. 7: Coverages in Over-provisioned Network

is shown in Fig. 7. INLP and IQP formulations again gives more importance to targets covered less number of times. As a consequence, there is no uncovered targets for INLP and IQP formulations. Due to abundance of available sensors, all the formulations tries to increase the number of 2-covered and 3-covered targets and INLP and IQP formulations exceeds the other approaches in doing so. Among the greedy algorithms, the Greedy Quadratic performs much better as it reduces the number of lower covered targets.

D. Sensor usage Analysis

Fig. 8(a) shows the percentage of sensor usage for the scenarios of Fig. 5(a). Percentage of sensors used gradually increases with the number of targets until all the cameras are activated. All the formulations perform almost similarly. Fig. 8(b) is the sensor usage diagram for scenarios in Fig. 5(b). As the number of sensors gradually increases, lesser percentage of available sensors are activated. Although the number of sensors used by all formulations are similar when the number of available sensors is lower, the situation changes with the increase in available sensors. With 50 to 115 sensors, sensor usage of Greedy Linear and Greedy Quadratic is less than others. However in terms of coverage, they were always outperformed by IQP and INLP.

Interestingly, both Fig. 8(a) and Fig. 8(b) shows a clear transition from under-provisioned network to over-provisioned network and the sensor usage phenomena also changes accordingly. In Fig. 8(a) when the number of targets exceeds 50 and in Fig. 8(b) when the number of sensors falls below 55, the networks become under-provisioned. We can see that all of our formulations used up almost all the sensors in such under-provisioned networks. As a result all the curves become almost linear and parallel to the x-axis in these regions. Reducing sensor usage only happened in the over-provisioned networks (i.e., the other side of the plots). In this region, although all the formulations used almost the same number of sensors as shown in Fig. 8(a), but INLP and IQP had much better coverage balancing over other formulations (see Fig. 5(a)). Among the variants of the greedy algorithm, Greedy Quadratic shows better coverage balancing over Greedy Linear, although

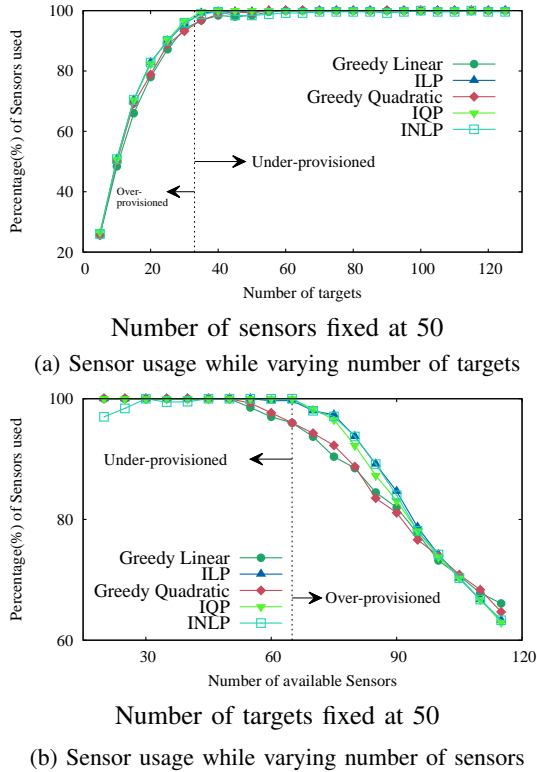


Fig. 8: Analysis of Sensors usage

they use the same number of sensors. From Fig. 8(b) and Fig. 5(b), we can draw a similar conclusion.

VII. RELATED WORKS

A large volume of research exists for the k -coverage problem where the researchers worked only with omnidirectional sensors [14] [8] [2] [11]. Li-Hsing Yen et al. [14] formulated an exact mathematical expression for the expected area that would be k -covered. Hafeeda and Bagheri [8] modelled k -coverage problem as optimal hitting set problem which is NP-hard [7]. Their proposed [8] k -coverage algorithm is inspired by the approximation algorithm in [4] and they suggested both centralized randomized k -coverage and distributed randomized k -coverage algorithms. In [2], the authors has worked with connectivity and coverage of WSN in 3D space. They proved that if there are k sensors with spherical sensing range in a Reuleaux tetrahedron then all the targets in that Reuleaux tetrahedron will be k -covered. Most of the works discussed so far assume that the locations are known. The algorithm presented by Yigal Bejerano in [3] can efficiently verify the k -coverage without any prior location information.

In [13], Tian and Georganas devised a node scheduling algorithm which would turn off redundant sensors without reducing the overall coverage of the network. This would be achieved by turning off only those sensors whose coverage region is covered by its neighbouring active sensors. In [11], Kumar et al. provided a solution to k -coverage problem where at any point of time most of the nodes are in sleep state. The authors study the lifetime of a network using different types

of node distributions such as $\sqrt{n} \times \sqrt{n}$ grid model, random uniform distribution and Poisson distribution.

There exist quite a few works using directional sensors. Jing and Abouzeid [1] formulate the coverage problem using directional sensors as Maximum Coverage using Minimum Sensors (MCMS) problem, and provide both centralized and decentralized greedy solutions. In [12], Munishwar and Abu-Ghazaleh present new algorithm which improves the greedy approaches of [1]. The optimal solution to k -coverage problem has been proved to be NP-hard by Fusco and Gupta in [6]. They also modelled the sensors to have a fixed viewing angle and overlapping pans. However, all of these works overlooked the coverage imbalance issue.

VIII. CONCLUSIONS AND FUTURE WORKS

Our work addresses a novel problem of coverage imbalance in k -coverage of VSN. Coverage imbalance is a serious problem in under-provisioned networks, where the networks does not have enough sensors to ensure k -coverage of all the targets. We extended the traditional ILP designed for single coverage and applied it to solve the multi-coverage problem. However, the extension does not balance the coverage of the targets. We further designed quadratic (IQP) and non-linear (INLP) version of the ILP that are capable of addressing the coverage balancing. IQP minimizes the vector distance between the attained and expected coverage. INLP maximizes the Balancing Index, \mathcal{BI} , which is the product of Fairness Index (\mathcal{FI}) and average coverage. As ILP, IQP, and INLP are not scalable for large problem instances, we developed a greedy approach, $CGkCA$ with two variants of incentive mechanism namely Greedy Linear and Greedy Quadratic. Even though both greedy approaches are outperformed by optimal algorithms, in under-provisioned networks Greedy Quadratic closely approximates the optimal solutions. We ran computer simulations to verify efficacy of the proposed formulations. In future, we plan to carry out more simulations using different scenarios, such as different grid sizes and different sensing ranges. We also plan to run $CGkCA$ under different incentive mechanisms and in real test beds.

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